Part III:
Line Drawings and Perception

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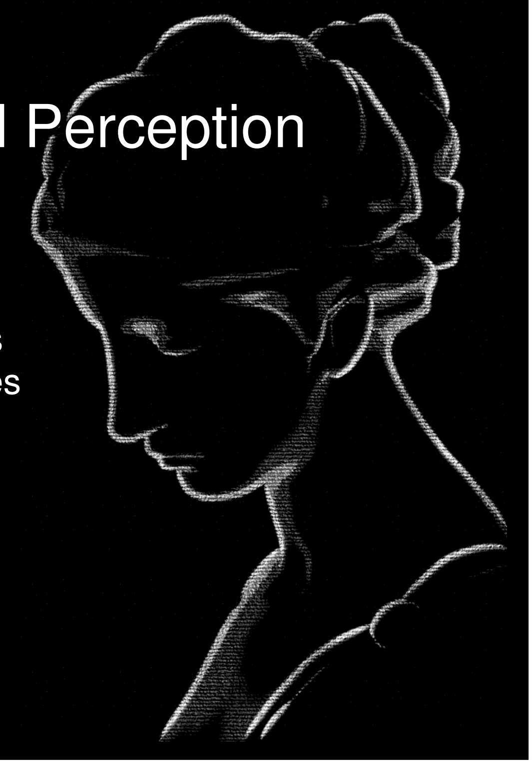
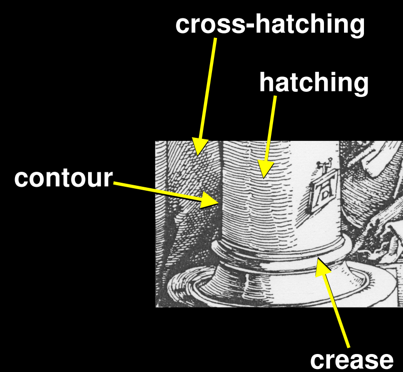
Line Drawings from 3D Models
SIGGRAPH 2005 Course Notes



Line drawings

Albrecht Dürer, “The Presentation in the Temple”
from *Life of the Virgin* (woodcut, circa 1505)



Line drawings bring together an abundance of lines to yield a depiction of a scene. This print by Dürer employs different types of lines that convey geometry and shading in a way that is compatible with our visual perception. We appear to interpret this scene accurately, and with little effort.

Some of the lines here, such as contours and creases, reveal only geometry. The fullness of this drawing comes from Dürer’s use of hatching and cross-hatching. These patterns of lines convey shading through their local density and convey geometry through their direction.

Line drawings

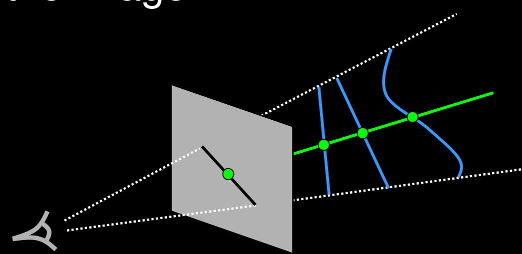
John Flaxman, “Venus Disguised Inviting Helen to the Chamber of Paris”
(illustration for *The Iliad*, etching, 1805)

Other drawings rely on little or no shading, such as this one by Flaxman. Here, the use of shading is limited to the cast shadows on the floor. The detail in the cloth is conveyed with lines such as contours and creases, and perhaps other lines such as suggestive contours, ridges and valleys. While artists can produce drawings like this, they don’t have access to the nature of the processes behind what they’re doing. They rely on their training, and use their own perception to judge the effects of their decisions.

Ambiguity of lines in images

The ambiguity of projection

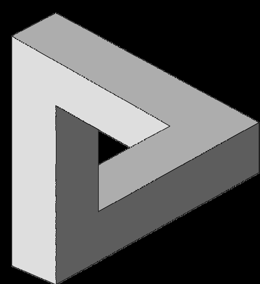
- an infinity of 3D curves can project to the same line in the image



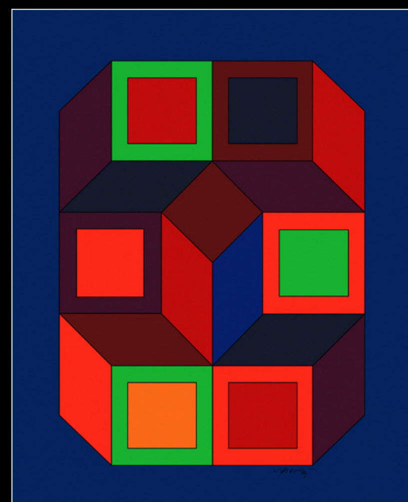
It’s actually a bit surprising that line drawings are effective at all. Upon first inspection, line drawings seem to be too ambiguous. An infinity of curves in 3D project to the same line in the image. All images have this ambiguity, but in photographs there are many other cues such as shading that help to indicate shape. Here we are looking just at individual lines.

But it turns out that individual lines contain a wealth of information about shape. This information is typically local in nature; our perception is somehow able to integrate all of this into a coherent whole. Well, sort of.

Impossible line drawings



The Penrose triangle (1958)

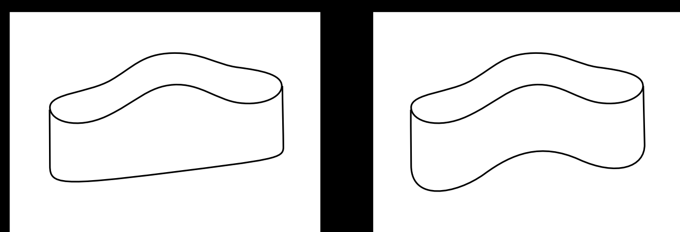


Victor Vasarely, XICO (silkscreen, 1973)

Line drawings of impossible 3D objects show us that this coherence is not global. The Penrose triangle, inspired by the work of M.C. Escher, is perhaps the simplest of the impossible figures. At first, it appears like an ordinary object. Closer inspection is a bit unsettling, and its inconsistencies are easily revealed, producing an non-convergent series of visual inferences (which can be quite fun to explore). Artists such as Vasarely have pushed this idea even further, producing vivid imagery that encourages us to explore several different inconsistent interpretations simultaneously.

Interactions between lines

- Interpretation of line drawings depends on context



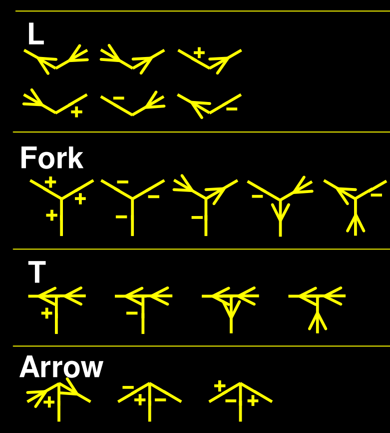
after [Barrow 1981]

Although you might think the Penrose triangle shows that there are no global effects for visual inference, it’s not that simple. The figure on the left appears to be raised in the center, while the figure on the right is flat on the top and bends along its length. Only the line along the bottom of the drawing differs. Nobody knows whether we perceive this difference because we integrate local information consistently, or perform certain types of non-local inference.

Interpretation of line drawings

Labeling of polyhedral scenes

- use an exhaustive catalog of possible labels of junctions
- determine configurations of consistent labels using constraint satisfaction
  [Waltz 1975]



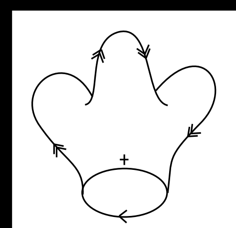
adapted from [Waltz 1975]

Use of non-local inference is plausible; algorithms exist for searching among the space of possibilities. Waltz's method line-labeling starts with catalogs of all possible line junctions—places where two or more lines meet. Shown here is the catalog of 18 junctions for classifying trihedral vertices in polyhedral scenes, where lines are labeled as convex (+), concave (-), or on a boundary (inside is to the right of the arrow). Then, methods for constraint satisfaction produce the set of all possible configurations for a particular picture. For an impossible figure, this set is empty.

Interpretation of line drawings

Labeling for more general scenes

- use larger catalogs of junctions
- rely upon methods to prune unreasonable or unlikely configurations
  [Barrow 1981, Malik 1987]



after [Malik 1987]

Methods for interpreting line drawings that contain smooth surfaces extend the junction catalogs and rely upon methods that prune away large numbers of unreasonable interpretations.

All of these methods label lines with a type; they don’t infer geometry. Furthermore, they are restricted to lines from contours and creases, and occasionally lines from shadows.

Interpretation of line drawings

A range of algorithms exist for interpreting certain types of line drawings

Not very much is known about how humans process line drawings

However, a lot is known about what **information** people could be using for interpretation

While these algorithms suggest that exhaustive search may be a viable method for scene interpretation, they don't say anything directly about how people interpret line drawings. In fact, not very much is known about that. Even so, we can still be very specific about what information is available in a line drawing. This is the information that our perceptual systems are probably using.

Interpretation of line drawings

Each line in a line drawing **constrains** the depicted shape

- The nature of the constraint depends on the type of line
- The type of the line can sometimes be inferred from context (within the drawing)

Ambiguity always remains, although some interpretations are more likely than others

Essentially, each line in a drawing places a constraint on the depicted shape. In the discussion that follows, we will examine the information that different types of lines provide. In the end, the answer is never unique. However, our perceptual systems excel at discovering the most likely interpretations.

Information in line drawings

- Lines can mark **fixed** locations on the shape
  - creases (sharp folds)
  - ridges and valleys
  - surface markings (texture features, material boundaries, …)
  - hatching lines (although density is lighting-dependent)
- Lines can mark **view-dependent** locations on the shape
  - contours (external and internal silhouettes)
  - suggestive contours
- Lines can mark **lighting-dependent locations** on the shape
  - isophotes (boundaries of attached shadows or in cartoon shading)
  - boundaries of cast shadows

First, we’ll consider lines that mark fixed locations on a shape. This includes creases, ridges and valleys, and surface markings.

Then, we’ll consider view-dependent lines. The most important is the contour, which lets us infer surprisingly rich information about the shape.

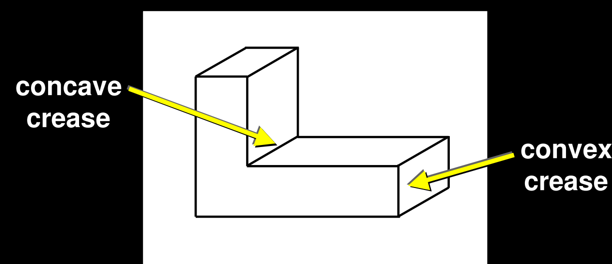
There are also lines whose locations are lighting-dependent, such as edges of shadows; these won’t be discussed here.

Of these, only creases and contours are well understood. Research on the information other types of lines provide is ongoing.

Information in creases

Creases mark discontinuities in surface orientation

– creases are either convex or concave (this cannot be determined using only local information)
– there is often a luminance discontinuity in shaded imagery



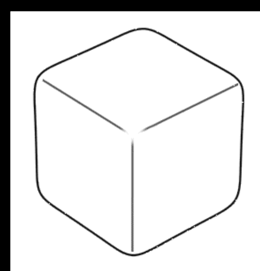
Creases mark orientation discontinuities, and are typically visible in a real image as a discontinuity in tone.

The crease can be concave or convex; local information doesn’t let us determine which—algorithms for line labeling only proceeded by considering all the possibilities, and then enforcing non-local consistency.

Information in ridges and valleys

Ridge and valley lines mark locally rapid changes in surface orientation

- one possible extension of creases to smooth surfaces
- like creases, they cannot be distinguished using only local information



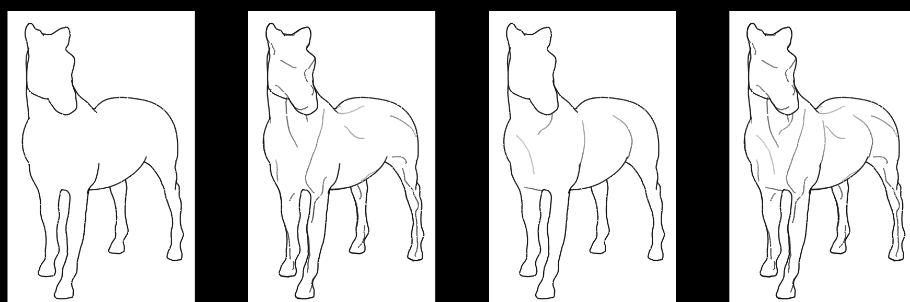
Ridges and valleys mark locally maximal changes in surface orientation. They are often visible in real images as sudden (but smooth) changes in tone.

The ridges on this rounded cube are particularly effective in conveying its shape.

Information in ridges and valleys

Their use is still unresolved

- many ridges and valleys seem to successfully convey shape
- others seem to convey surface markings (inappropriately)
- viewers can locate them in shaded imagery [Phillips 2003]



contours **with ridges** **with valleys** **with both**

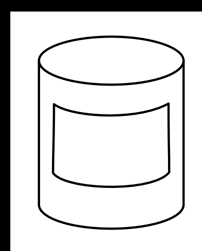
Research on the use of ridges and valleys in line drawings is ongoing. When used alongside contours, ridges and valleys can often produce an effective rendering of a shape; the valleys on the side of the horse are successful. In other cases, they look like markings on the surface of the shape, such as the ridges on its head.

They are reasonable candidates for line drawings, as there is psychological evidence that viewers can reliable locate ridges and valleys in shaded imagery.

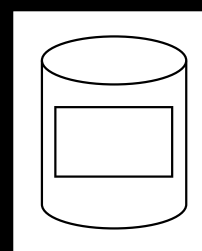
Information in surface markings

Can be located arbitrarily

- but seem to convey shape when they lie along geodesics (locally shortest paths on the surface)
  [Stevens 1981, Knill 1992]
- related to perception of texture
  [Knill 2001]



along geodesics



not along geodesics

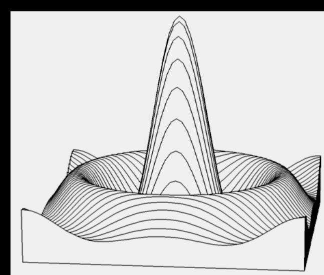
Markings on a surface can appear as arbitrary lines inside the shape. However, for a certain type of line known as a geodesic, they can also convey shape. (Geodesics are lines on the surface that are locally shortest paths.)

Stevens points out that for many fabricated objects, surface markings are commonly along geodesics. For a more general class of surfaces, Knill draws connections between texture patterns and sets of parallel geodesics.

Information in surface markings

Parallel lines in space can also convey shape

- building correspondences between adjacent lines (using tangents of the curves) lets the viewer infer the shape
  [Stevens 1981]

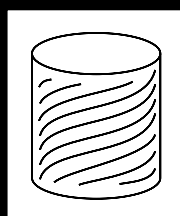


When used in repeating patterns, other curves can be effective as well. Sets of parallel lines, which are often used to construct plots of 3D functions, are one notable example. The images that result are analogous to using a periodic solid texture. Stevens points out that all one needs to do to infer the shape is to build correspondences between adjacent lines, matching up points with equal tangent vectors.

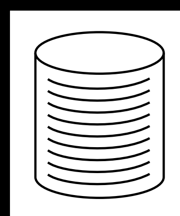
Information in hatching

Conveys shape through direction of hatching lines

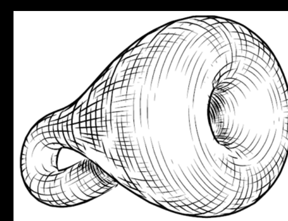
- more effective when drawn along geodesics
  [Stevens 1981, Knill 2001]
- lines of curvature are particularly effective
  [Girshick 2000, Hertzmann 2000]



along geodesics



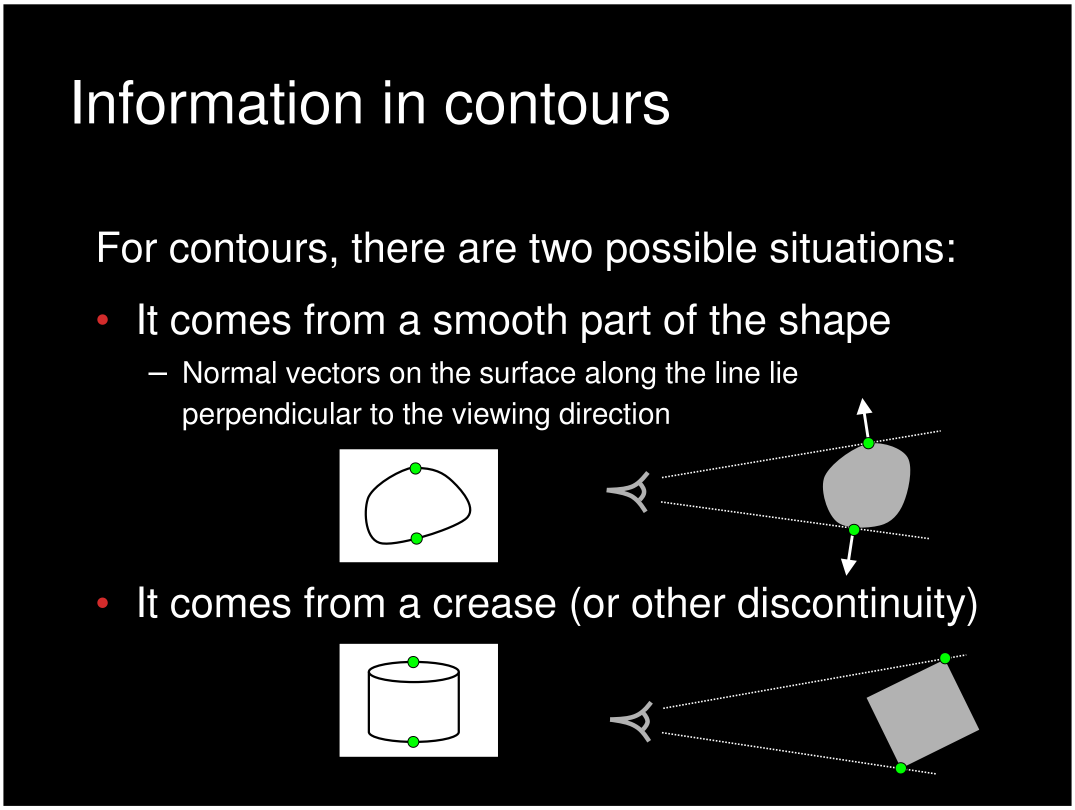
along lines of curvature



[Hertzmann 2000]

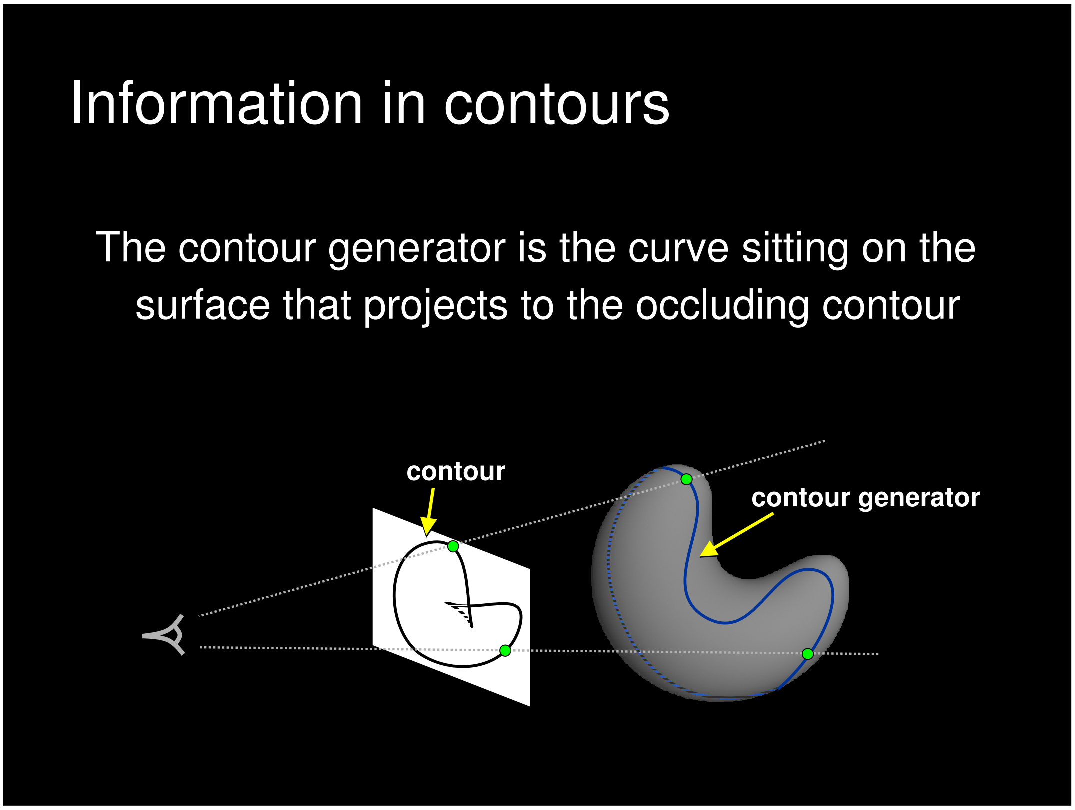
The use of repeating patterns of lines forms the basis of hatching. These lines convey shape in two different ways; they convey shape directly when they are drawing along geodesics. And they convey shape indirectly through careful control of their density, which can be used to produce a gradation of tone across the surface. Particularly effective renderings are obtained when lines of curvatures are used, which are lines that align with the principal curvature directions, and also happen to be geodesics. To convey tone, these lines are used in careful combinations that control their density in the image.

This concludes the discussion on lines whose locations are fixed on the shape.



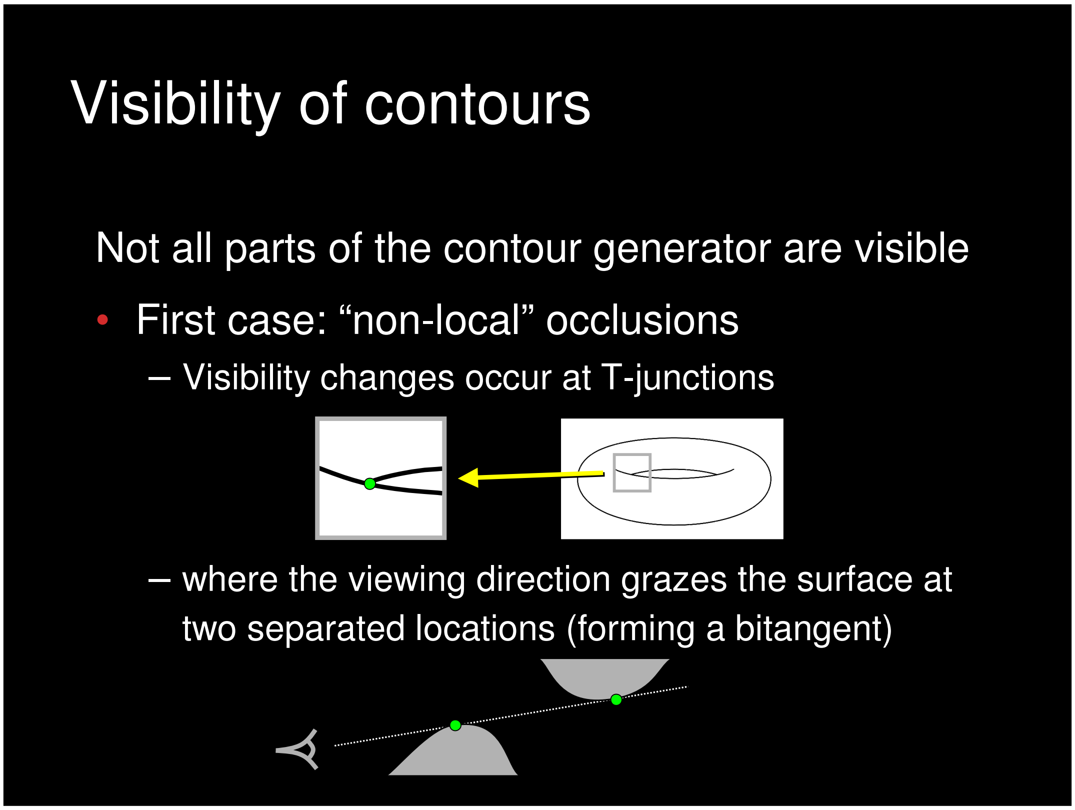
Next comes lines whose location on the shape depends on the viewpoint.

The contours are the most notable example of such lines. There are two situations when contours are formed. On a smooth surface, contours are produced when the surface is viewed edge-on. On an arbitrary surface, contours can also be produced along a crease.



In either case, sitting on the surface is a 3D curve known as the contour generator. This curve marks all local changes in visibility across the shape. For a generic (non-singular) viewpoint, the contour generator consists of a set of isolated loops.

The contour generator projects into the image to become the contour. Not all parts of the contour are visible.



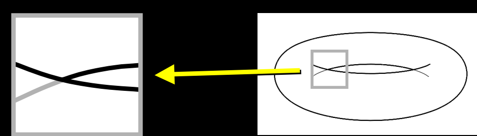
Let’s consider the different cases of visibility for contours.

On a smooth surface, the first case is when one part of the shape occludes another more distant part. This appears in the image as a T-junction, where the contour goes behind another part of the shape. At the location where the visibility changes, the visual ray is tangent to the surface in two places.

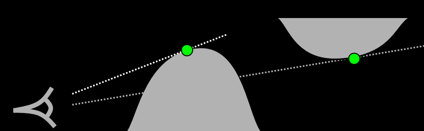
Visibility of contours

Not all parts of the contour generator are visible

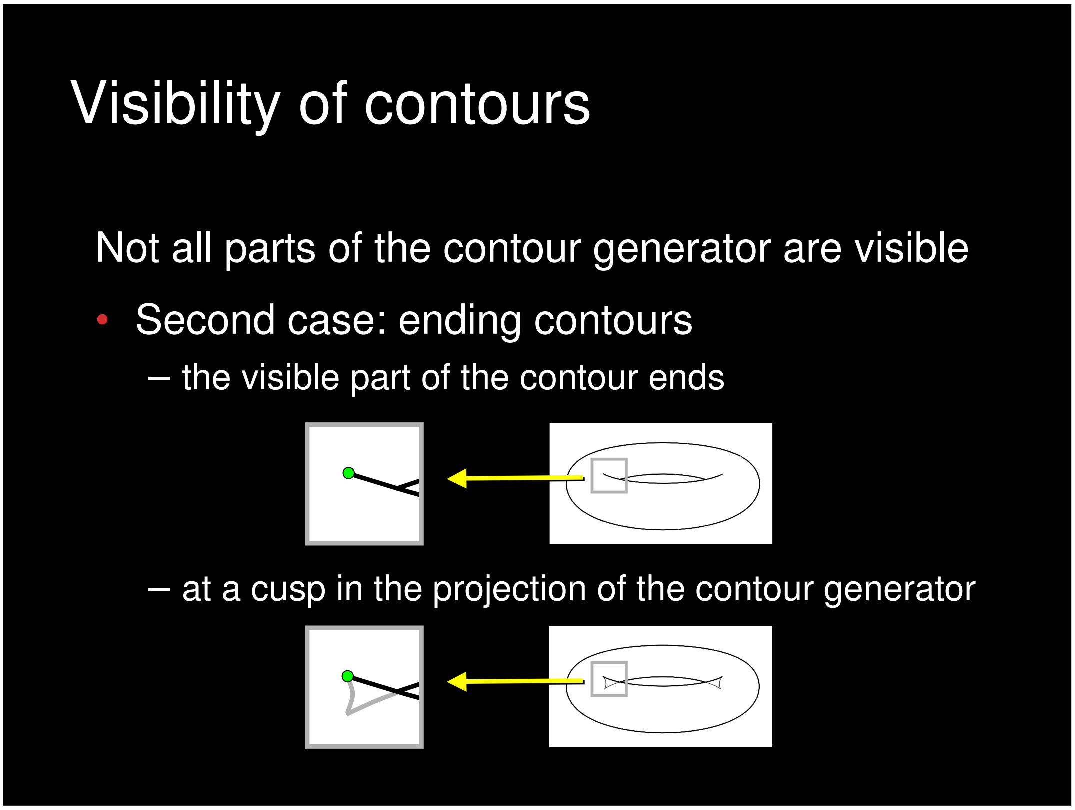
- First case: “non-local” occlusions
  - The contour continues in back



  - and is simply occluded



The contour then continues behind the shape, and is occluded. It is seen here in this transparent line-drawing of a torus.

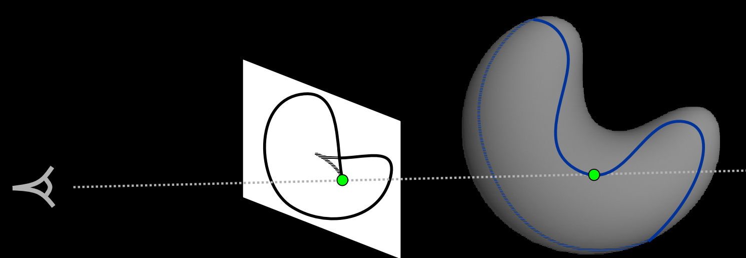


The second case occurs where the contour comes to an end in the image. When the occluded part of the contour continues, it does so at a cusp in the contour.

Visibility of contours

Not all parts of the contour generator are visible

- Second case: ending contours
  - the tangent of the contour generator lines up with the viewing direction at this point

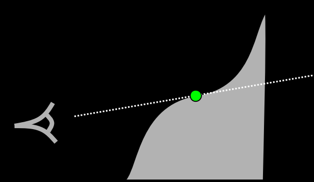


This cusp occurs because the contour generator lines up with the viewing direction, so that its tangent projects to a point.

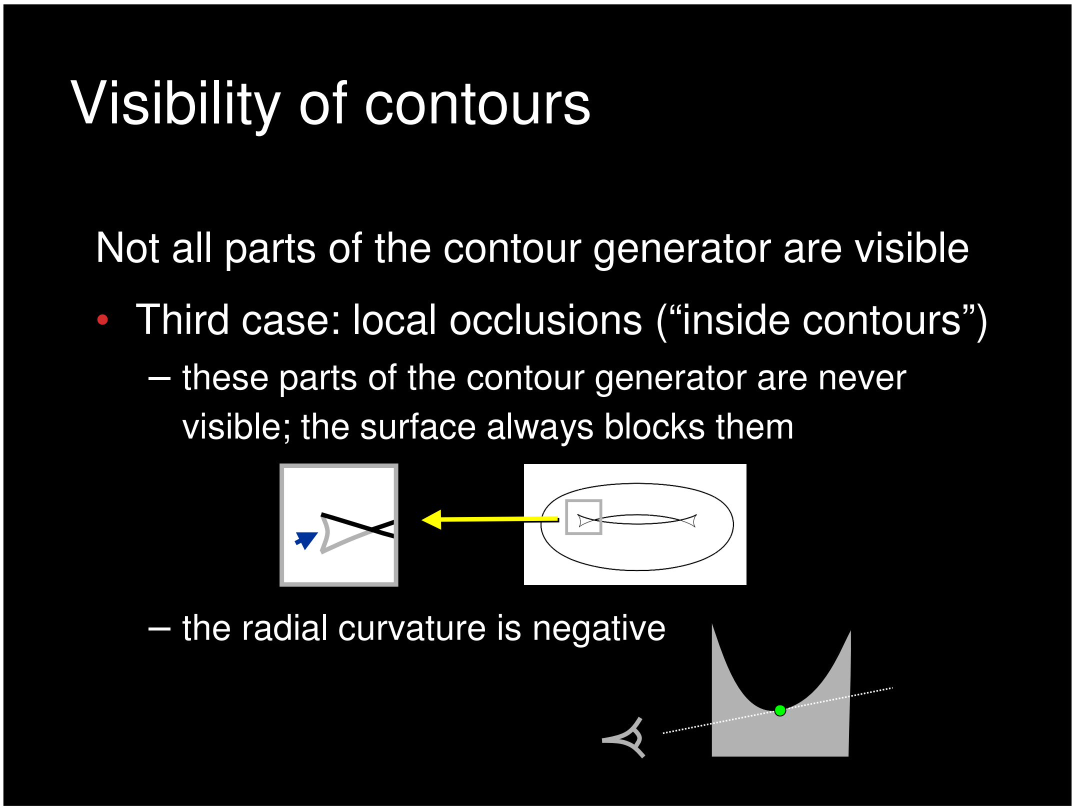
Visibility of contours

Not all parts of the contour generator are visible

- Second case: ending contours
  - the radial curvature is zero (the viewing direction is an asymptotic direction of the surface)



At an ending contour, the radial curvature is zero, which means that we’re looking along an inflection—an asymptotic direction of the surface.



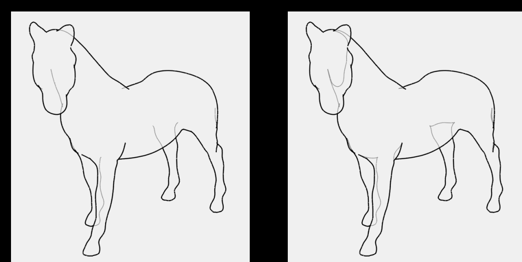
The last case is a local occlusion; places where the surface has no choice but to occlude itself.

These are locations where the radial curvature is negative.

Visibility of contours

Not all parts of the contour generator are visible

- Third case: local occlusions (“inside contours”)
  - these contours can be confusing in transparent renderings

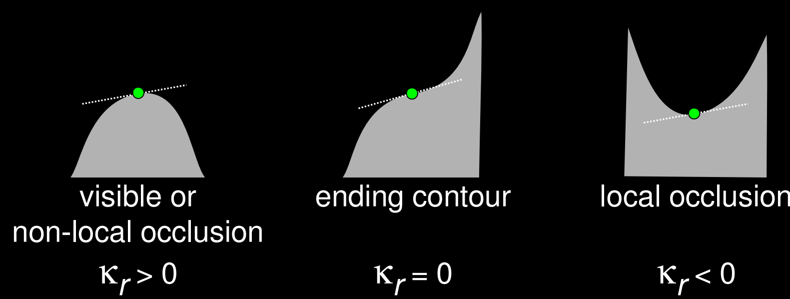


In transparent renderings of contours, one typically does not draw the local occlusions (which are identified by having negative radial curvature), as the results can be confusing.

These curves actually correspond to regular contours for an inside-out version of the surface.

Visibility of contours

The three cases (on smooth surfaces):

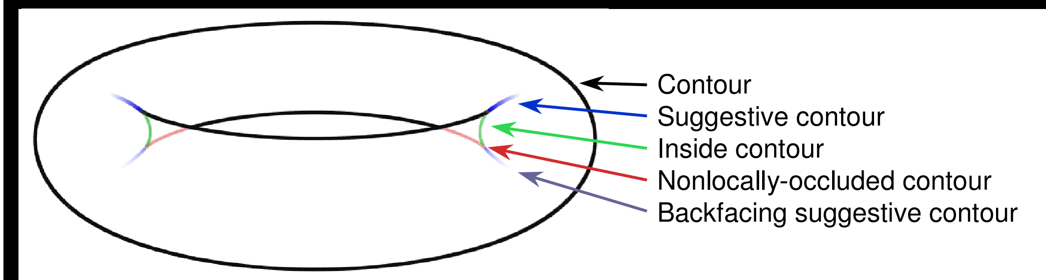


Here are the three cases, all together.

Contours and suggestive contours

Recall: suggestive contours can extend contours

- Extend ending contours at cusps
- Backfacing suggestive contours extend nonlocally-occluded contours

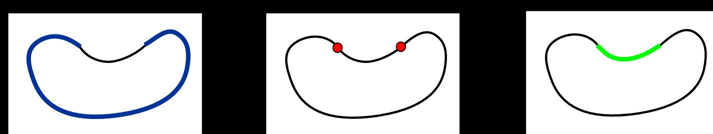


It's worth noting that suggestive contours extend true contours at the ending contour cusps, and that backfacing suggestive contours always extend nonlocally-occluded (hidden) contours. In other words, the lines Do The Right Thing.

Apparent curvature of contours

The apparent curvature κ_{app} is the curvature of the contour in the drawing (or image)

- for outward-pointing normal vectors…

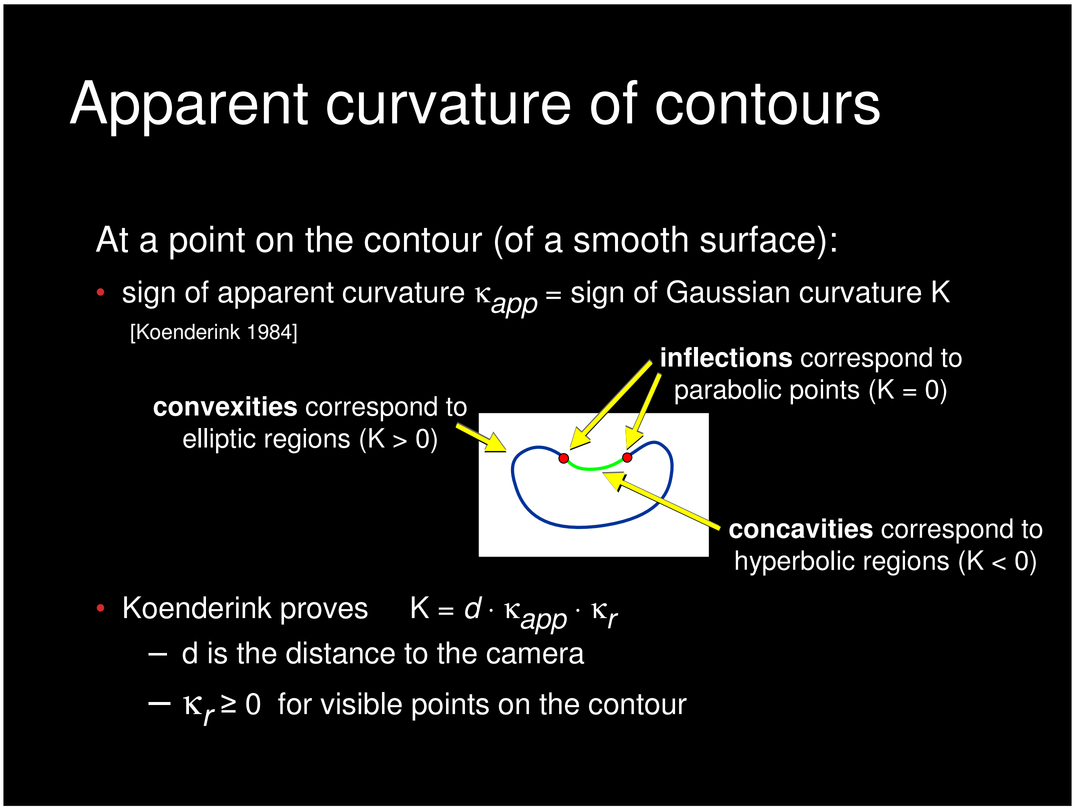


$\kappa_{app} > 0$ $\kappa_{app} = 0$ $\kappa_{app} < 0$

- At the cusp of an ending contour, κ_{app} is infinite

Now, let's consider what the contours look like in the image.

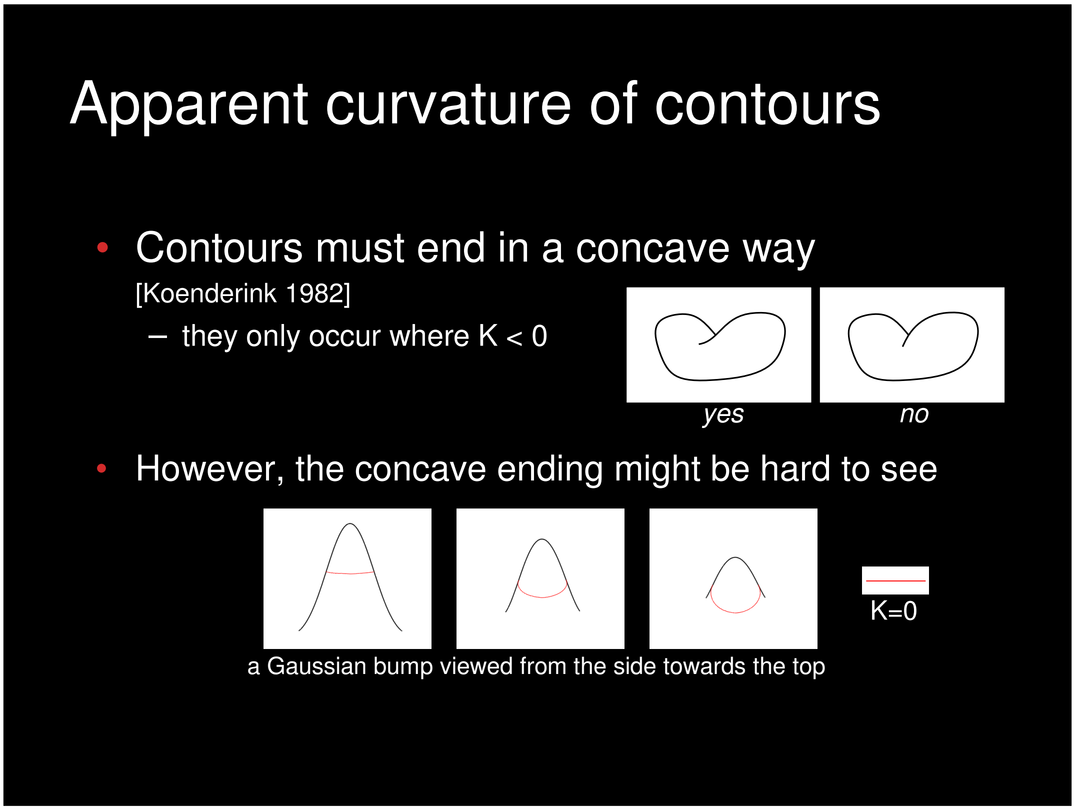
The apparent curvature is simply the curvature of the contour in the drawing. When working with outward-pointing normal vectors, the convex parts of the contour have positive apparent curvature, the concave parts have negative apparent curvature, and it's zero at the inflections. At the ending contours, the apparent curvature is infinite due to the cusp.



Koenderink proved a surprising and important relationship between the apparent curvature and the Gaussian curvature. Specifically, for visible parts of the contour on a smooth surface, they have the same sign. This means we can infer the sign of the Gaussian curvature simply by looking at the contour.

Koenderink gives a formula that connects these two quantities, that also involves the distance to the camera (this is because the apparent curvature gets larger as the object is farther away) and the radial curvature.

Note that because the radial curvature is never negative for visible parts of the contour, this allows us to infer the sign of the Gaussian curvature.



A related result is that since ending contours only occur on hyperbolic regions (where K is negative), the contours must end in a concave way—approaching their end with negative apparent curvature.

Even so, in many cases, this concave ending is difficult to discern, as is the case for a Gaussian bump when viewed from above.

Koenderink and van Doorn also noticed that artists tend to draw lines that are missing these concave endings.

Information in suggestive contours

Relation to contours

- can connect to ending contours
  (they line up in the image) [DeCarlo 2003]
  – makes it hard to tell where contour ends

Contours are typically easily to detect in real images, at least when the lighting is right. And there are many studies that demonstrate how people use them for visual inference.

Suggestive contours are another type of line to draw, and whether they are in fact detected and represented by our perceptual processes is still an open question. They do seem to produce convincing renderings of shape in many cases.

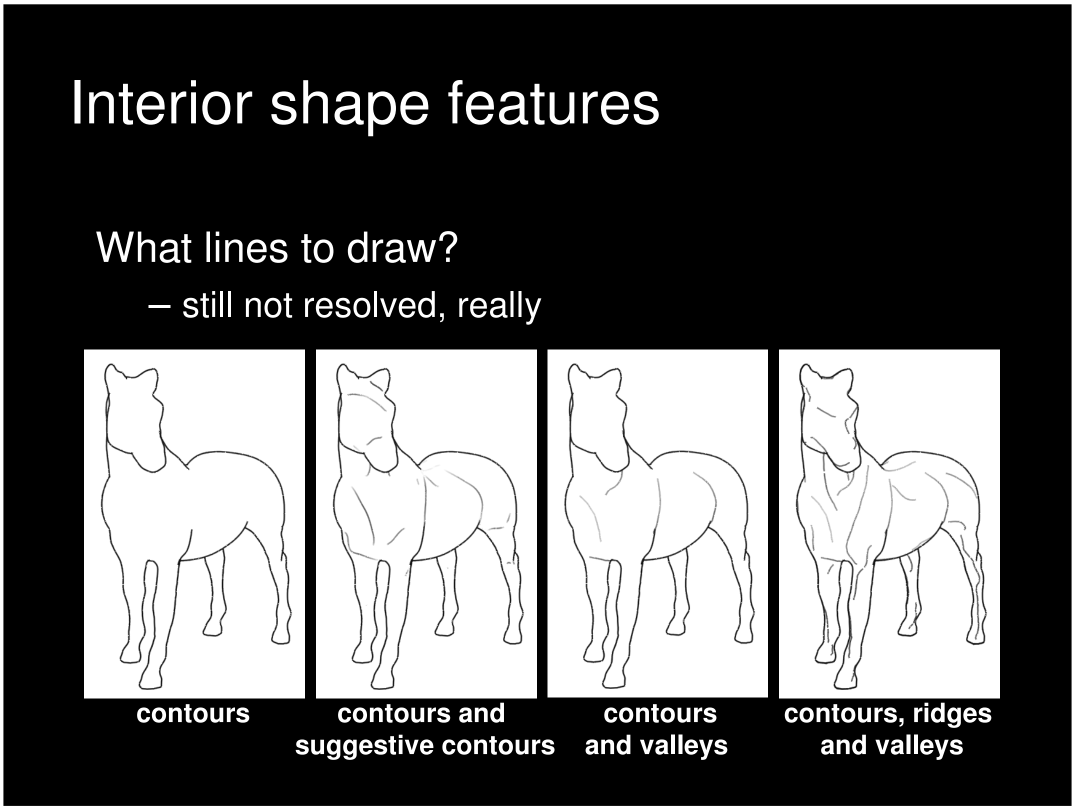
The fact that suggestive contours are an exaggeration of contours to account for nearby viewpoints is encouraging, as is their property that they smoothly line up with contours in the image.

Information in suggestive contours

Can only appear in hyperbolic regions ($K < 0$)

- often approach parabolic lines (where $K = 0$) away from the contour
  [DeCarlo 2004]

We can say something about what information they provide. We know that in many cases that the suggestive contours approach the parabolic lines away from the contour. They approach it from the hyperbolic side, as suggestive contours can only appear where K is negative, as there are no directions with zero curvature where K is positive. We hope to be able to say more about this in the future.

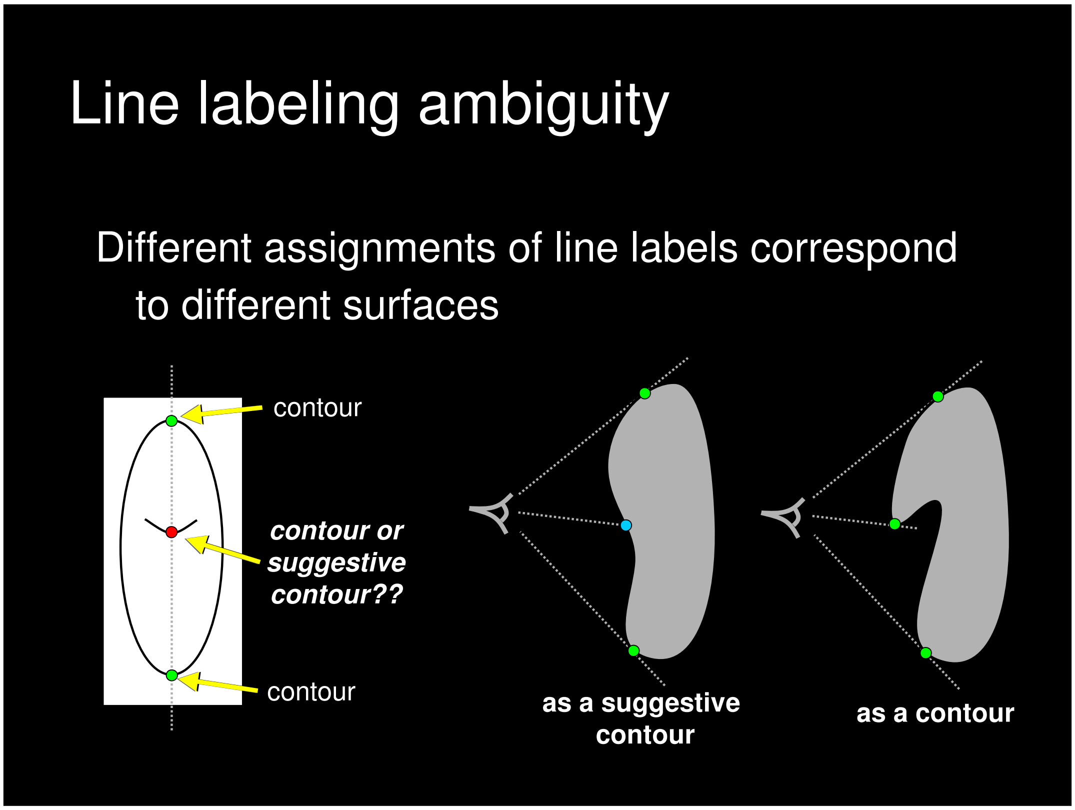


We can compare renderings with ridges and valleys to renderings with suggestive contours.

On the horse from this viewpoint, the rendering with just valleys is actually quite convincing. As noted earlier, many of the ridges appear as surface markings here.

For the valley rendering, some features are missing, but the more salient features on the side of the horse are depicted. Note that the slight differences between the lines from suggestive contours, and from valleys. The shapes they convey do appear to be a little different. Clearly there is a lot of interesting work to do here.

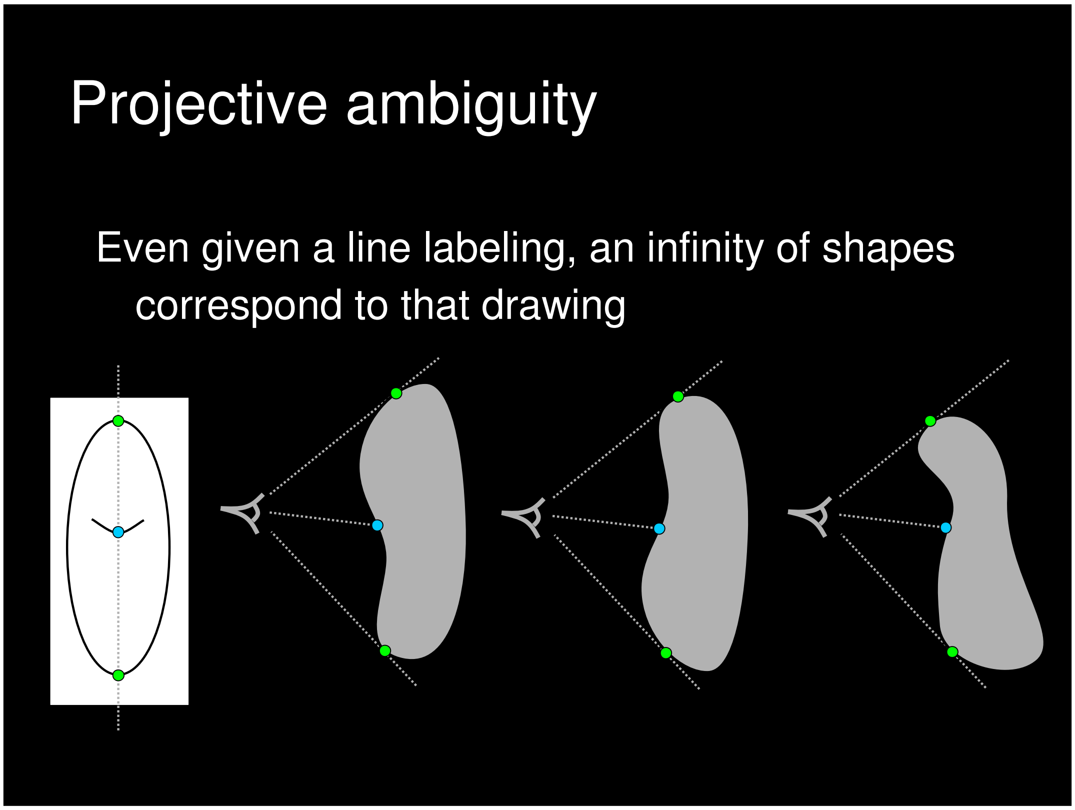
This concludes our discussion of what information particular lines provide.



Of course, this information can only be used if we know the types of the lines when we're given a drawing. Earlier we discussed algorithms for line drawing interpretation; approaches like this are reasonable to consider for this purpose.

But even if we do use these algorithms, there are often several different labelings that are consistent. Given the line drawing on the left which depicts an elliptical shape with a bump, we will successfully be able to label the green points as contours. The red point, however, can be either a contour or suggestive contour. Two possible shapes that match these labelings are shown on the right.

Presumably this problem cannot be solved in general; there will always be ambiguity. It's possible that when artists make line drawings, they are careful to shape the remaining ambiguity so it won't be a distraction. These are difficult problems.



And even with a line labeling, there is the ambiguity of projection. At first, this seems hopeless. Yet sketching interfaces like Takeo Igarashi's Teddy seem to be quite successful by using "inflation". How can this be? Well, there are reasonable constraints on smoothness that we can expect of the underlying shape. We also presume that the artist has drawn all of the important lines, so that no extra wiggles remain.

These issues are the source of one crucial challenge for sketch-based shape modeling.

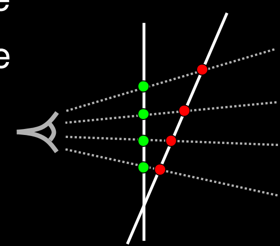
Bas-relief ambiguity

The projective ambiguity that preserves planarity

– ambiguity in (Lambertian) shaded imagery
[Belhumeur 1999]

– preserves contours, shadow boundaries, (relative) signs of curvature

– has perceptual significance
[Koenderink 2001]



We can be even more specific with regard to this ambiguity. It seems that even for real images, there are well defined ambiguities for particular types of imagery. One notable example is the ambiguity that remains when viewing a shape with Lambertian illumination. It turns out that there is a group of shape distortions that can be applied to a shape that, with an corresponding transformation of the lighting positions, produce the same image (approximately).

The shape distortions here are produced by a three-dimensional mapping known as the generalized bas-relief transformation. As shown here, this is the mapping that moves points along visual rays and also preserves planes. It also preserves contours, boundaries of shadows, and the relative signs of curvature on the shape.

Perhaps most interestingly is that when you ask people to describe the shapes they see in shaded imagery, they answer consistently only up to this ambiguity transformation. How they answer within this space of possibilities depends on how you ask them.

Evaluation of line drawings

How can we tell whether a line drawing is accurately perceived?

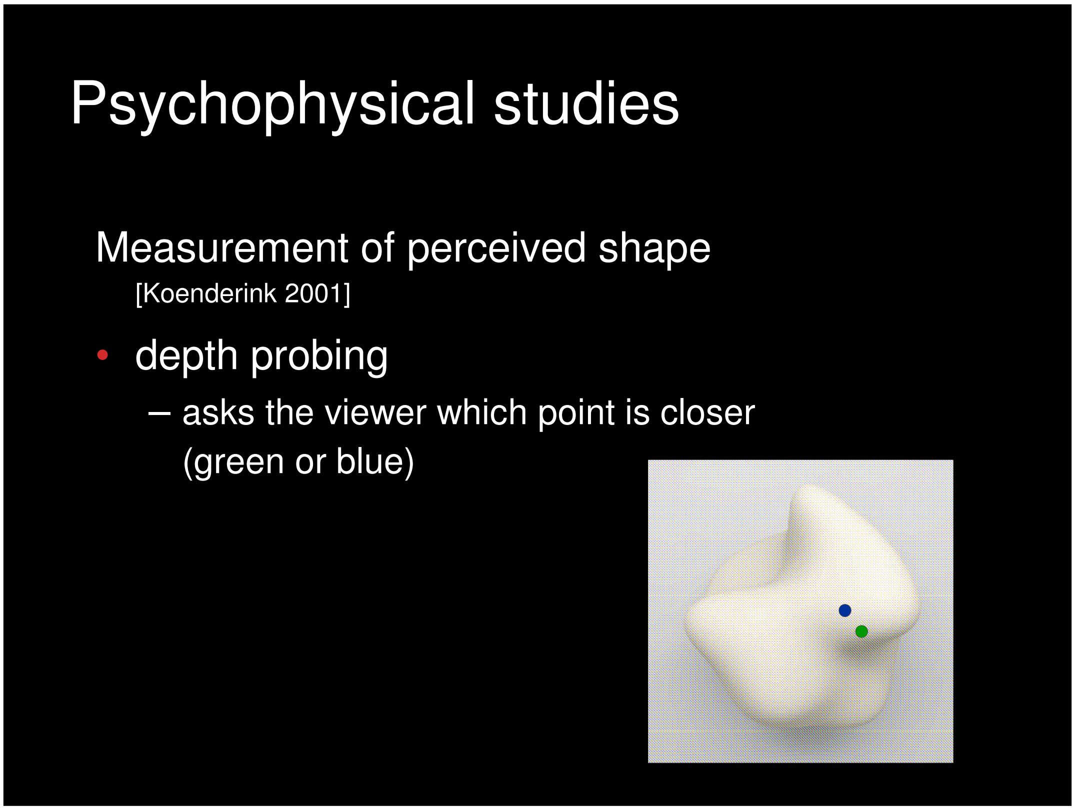
- compare it to drawings by skilled artists
- psychophysical measurement
  - resulting shape should be consistent with the shape used to construct the drawing (modulo ambiguity)
  - one study on a hand-made line drawing suggests the bas-relief ambiguity could be appropriate here [Koenderink 1996]

So how can we be sure that a line drawing we make is perceived accurately?

Well, one way is to compare that line drawing to one made by a skilled artist. This is difficult and subjective.

Another way, based in psychophysics, is to simply ask the viewer questions about the shape they see. If this is done right, you can reconstruct their percept and compare it to the original shape. When this comparison is done, it should be with respect to the appropriate ambiguity transformation.

Koenderink and colleagues have already performed a study like this on a single line drawing (and compared it to other depictions, such as shaded imagery). Their results suggest that the bas-relief ambiguity might be the appropriate ambiguity transformation to use here.



So what kinds of questions can you ask viewers?

In psychophysics, the answer is typically – very simple ones, and lots of them.

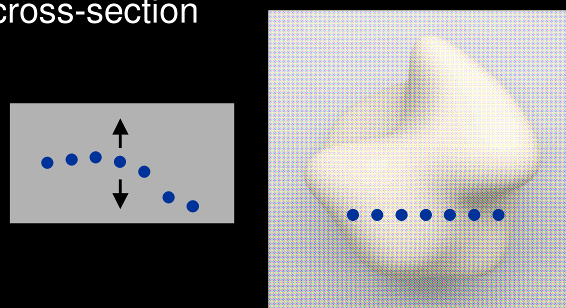
Koenderink and colleagues describe a set of psychophysical methods for obtaining information about what shape a viewer perceives.

The first they describe is called relative depth probing. The viewer is shown a display like this one, and simply asked which point appears to be closer. They are asked this question for many pairs of points.

Psychophysical studies

Measurement of perceived shape
[Koenderink 2001]

- depth profile adjustment
  - viewer adjusts points until they match the profile of a particular cross-section

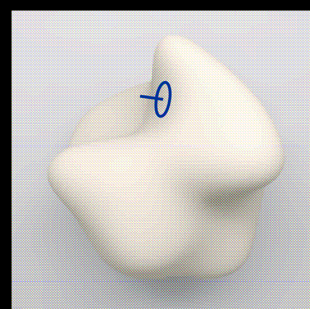


Another method is known as depth profile adjustment. Here, the viewer adjusts points (vertically) to match the profile of a particular marked cross-section on the display.

Psychophysical studies

Measurement of perceived shape
[Koenderink 2001]

- gauge figure adjustment
  – viewer adjusts a disk until it appears to sit on the tangent plane of the surface



Their third method is known as gauge figure adjustment. Here, the viewer uses a trackball to adjust a small figure that resembles a thumbtack, so it looks like its sitting on the surface.

All of these methods are successful. But gauge figure adjustment seems to give the best information given a fixed number of questions.